

Models for Light-Cone Meson Distribution Amplitudes

Patricia Ball and Angelique N. Talbot

IPPP, Department of Physics, University of Durham, Durham DH1 3LE, UK

E-mail: Patricia.Ball@durham.ac.uk, A.N.Talbot@durham.ac.uk

ABSTRACT: Leading-twist distribution amplitudes (DAs) of light mesons like π, ρ etc. describe the leading nonperturbative hadronic contributions to exclusive QCD reactions at large energy transfer, for instance electromagnetic form factors. They also enter B decay amplitudes described in QCD factorisation, in particular nonleptonic two-body decays. Being nonperturbative quantities, DAs cannot be calculated from first principles, but have to be described by models. Most models for DAs rely on a fixed order conformal expansion, which is strictly valid for large factorisation scales, but not always sufficient in phenomenological applications. We derive models for DAs that are valid to all orders in the conformal expansion and characterised by a small number of parameters which are related to experimental observables.

KEYWORDS: Meson Distribution Amplitudes, QCD Factorisation, B Physics.

Contents

1. Introduction	1
2. Definitions and Conformal Expansion	3
3. Models for ϕ_π and ϕ_K	5
4. Results for $f_+^{B \rightarrow \pi}(0)$	11
5. $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ in QCD Factorisation	12
6. Summary and Conclusions	16

1. Introduction

Light-cone distribution amplitudes (DAs) appear in the description of hard exclusive QCD processes by factorisation, schematically

$$\text{amplitude} \sim \prod_{j,i} \phi_{\text{out},j}(n_j) \otimes T(Q^2; n_j, n_i) \otimes \phi_{\text{in},i}(n_i), \quad (1.1)$$

where the labels i and j refer to the hadrons in the incoming and outgoing states, respectively, $\phi(n)$ is the DA that describes the probability amplitude for a hadron to be found in the parton state n and $T(Q^2; n_i, n_j)$ is the perturbative function that describes the hard scattering between the partons; Q^2 is the hard momentum transfer. The symbol \otimes indicates a convolution, i.e. a sum or integral over the parton degrees of freedom that correspond to the states n_i and n_j . Factorisation implies that the process can be split into two separate regimes, the hard-scattering subprocess T , which involves a short-distance momentum transfer and can be calculated in perturbation theory, and the long-distance hadronisation of the partons emerging from the short-distance process, described by the process-independent DAs ϕ . The expansion of the amplitude in (1.1) is ordered by inverse powers of the momentum transfer Q^2 , which corresponds to a light-cone expansion of the process in terms of contributions of increasing twist. In this paper we are concerned with the leading-twist DA of light mesons. Although we shall focus on the pseudoscalar mesons π and K , the models we suggest are equally well applicable to — and in fact have already [1] been applied to — vector mesons (ρ , ω , K^* and ϕ).

The leading-twist meson DA of a meson is related to its Bethe-Salpeter wave function by integrating out the dependence on the transversal momentum k_\perp ,

$$\phi(u) \sim \int_{k_\perp^2 < \mu^2} d^2 k_\perp \phi(u, k_\perp),$$

where u is the longitudinal momentum fraction carried by the quark (and $\bar{u} \equiv 1 - u$ that carried by the antiquark). Originally introduced in the context of hadron electromagnetic form factors or the pseudoscalar-photon transition form factor [2, 3], light-meson DAs have, in recent years, attracted increasing interest also in B physics due to their appearance in QCD sum rules on the light-cone [1, 4, 5, 6] and factorisation formulas for B decay amplitudes [7] and form factors [8]. The theory of meson DAs is well understood [9, 10, 11] and suggests their parameterisation in terms of a partial wave expansion in conformal spin. One advantage of this expansion is that the contributions of higher conformal spins to the convolution integral and hence the physical amplitude are suppressed by the highly oscillating behaviour of the corresponding partial waves, another one that conformal symmetry ensures, to leading-logarithmic accuracy, multiplicative renormalisation of each partial wave. The combination of both features suggests the construction of models for DAs based on a truncated conformal expansion, where only the first few waves are included, typically one to three. Not much is known, in general, about the amplitudes of these partial waves. For the π and η some experimental information on the $\pi(\eta)\gamma\gamma^*$ transition form factor is available [12], which one can use to extract values of the first two amplitudes. From the theoretical side, there exist a few dated lattice calculations for the second moment of the π DA [13], and a recent retry [14]; unfortunately, these results are still preliminary and cannot yet be used in phenomenological applications. Other theoretical calculations, for both pseudoscalar [9, 10, 15, 16] and vector mesons [11, 17], use the method of QCD sum rules, which turns out to be not very suitable for higher moments; we will come back to that point in Sec. 3.¹ It is probably fair to say that at present some information is available on the lowest moments of the π DA, but much less so for other mesons. In view of the growing demands on B physics to deliver “precision results”, cf. e.g. Ref. [20], and the prominent role of QCD factorisation in testing the mechanism(s) of flavour violation, it then appears timely to assess the actual theoretical uncertainty of hadronic decay amplitudes induced by the truncation of the conformal expansion and to devise alternative models for light-meson DAs that do not rely on conformal expansion, but establish a closer connection between the characteristics of DAs accessible in “classical” applications of pQCD in hard exclusive processes and their use in B physics. This is precisely what we aim to achieve in this paper.

Our models for the leading-twist DAs of the π and K are based on the fall-off behaviour of the n th Gegenbauer moment of these amplitudes, a_n , in n , which we assume to be power-like. We shall argue that such a behaviour can be justified for instance from the known perturbative contributions to the a_n . We formulate our models in terms of a few parameters, notably the first inverse moment of the DA, which, for the π , is directly related to experimental data, and the strength of the fall-off of the a_n . The models can be summed to all orders in the conformal expansion and give predictions for the full DA at a certain scale. Depending on the parameters, the models also predict a nonstandard behaviour of the DAs at the endpoints, which affects for instance the scaling, in m_b , of the $B \rightarrow$ light meson form factors at zero momentum transfer.

Our paper does *not* aim to give a fully-fledged analysis incorporating all available

¹These problems have been evaded in Refs. [18, 19] in a modified version of QCD sum rules using nonlocal condensates.

constraints on the π DA from low-energy experimental data as scrutinised in Refs. [19, 21, 22]. Rather, we aim to work out the gross features of leading-twist DAs, which are likely to apply to all light pseudoscalar and vector mesons.

Our paper is organised as follows: in Sec. 2 we define the leading-twist DA of pseudoscalar mesons and discuss its partial-wave expansion in conformal spin. In Sec. 3 we derive models for the DA based on the fall-off behaviour of higher partial-waves and formulate constraints on the model parameters. In Sec. 4 we investigate the dependence of one important quantity in B physics, the semileptonic form factor $f_+^{B \rightarrow \pi}(0)$, on the model DAs, and in Sec. 5 we study their effect on CP asymmetries and branching ratios in nonleptonic B decays. We summarise and conclude in Sec. 6.

2. Definitions and Conformal Expansion

Distribution amplitudes are defined in terms of matrix elements of nonlocal operators near the light-cone. For pseudoscalar mesons P in particular one has

$$\begin{aligned} \langle 0 | \bar{q}_1(x) \gamma_\mu \gamma_5 [x, -x] q_2(-x) | P \rangle = & i f_P p_\mu \int_0^1 du e^{i\xi p x} \left[\phi_P(u) + \frac{1}{4} m_P^2 x^2 \mathbb{A}_P(u) + O(x^4) \right] \\ & + \frac{i}{2} f_P m_P^2 \frac{1}{p x} x_\mu \int_0^1 du e^{i\xi p x} [\mathbb{B}_P(u) + O(x^2)], \end{aligned} \quad (2.1)$$

where $\xi = 2u - 1$ and we use $[x, y]$ to denote the Wilson-line connecting the points x and y :

$$[x, y] = \text{P exp} \left[ig \int_0^1 dt (x - y)_\mu A^\mu(tx + (1 - t)y) \right]. \quad (2.2)$$

In Eq. (2.1), ϕ_P is the leading twist-2 DA, whereas \mathbb{A}_P and \mathbb{B}_P contain contributions from higher-twist operators. The corresponding definitions of vector meson DAs can be found in Ref. [10].

The extraction of the leading behaviour of the matrix elements on the light-cone yields ultraviolet divergences, whose regularisation generates a nontrivial scale-dependence that can be described by renormalisation group methods [2, 3]. Conformal invariance of QCD allows one to express the DA in terms of a partial wave expansion, also called conformal expansion, in contributions from different conformal spin, which do not mix with each other under a change of scale. This is true to leading logarithmic accuracy, but no longer the case at higher order, as the underlying symmetry is anomalous. For the leading-twist DA $\phi(u)$, for both pseudoscalars and vectors, the conformal expansion is in terms of Gegenbauer polynomials $C_n^{3/2}$,

$$\phi(u, \mu^2) = 6u(1 - u) \sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u - 1). \quad (2.3)$$

The coefficients a_n , the so-called Gegenbauer moments, renormalise multiplicatively to leading logarithmic accuracy:

$$a_n(Q^2) = a_n(\mu^2) \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{\gamma_0^{(n)}/(2\beta_0)} \quad (2.4)$$

with $\beta_0 = 11 - (2/3)n_f$. The one-loop anomalous dimension is given by [23]

$$\gamma_0^{(n)} = 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{(n+1)(n+2)} \right). \quad (2.5)$$

For π , ρ , ω , η , η' and ϕ , G-parity ensures that $a_{\text{odd}} = 0$ and that the DA is symmetric under $u \leftrightarrow 1-u$, whereas for K and K^* the nonzero values of a_{odd} induce an antisymmetric component of the DA. $a_0 \equiv 1$ is fixed by normalisation,

$$\int_0^1 \phi(u, \mu^2) = 1,$$

whereas all other a_n are intrinsically nonperturbative quantities. As they do not mix under renormalisation, Eq. (2.3) is well suited to construct models for ϕ : truncating the series after the first few terms, typically three, yields a parameterisation of the DA that is “stable” under a change of scale, except for the numerical values of a_n . Despite there being no small expansion parameter in the game, such a *truncated conformal expansion* is often – but not always – a meaningful approximation to the full DA, as we shall see below.

As the anomalous dimensions are positive (except for $\gamma_0^{(0)} = 0$), the contributions of higher a_n get suppressed for large scales, and for $\mu^2 \rightarrow \infty$ the DA approaches the so-called asymptotic DA

$$\phi_{\text{as}}(u) = 6u(1-u). \quad (2.6)$$

For many processes involving DAs, in particular in B physics, it is usually argued that a truncated conformal expansion be sufficient for the calculation of physical amplitudes as long as the perturbative scattering amplitude is “smooth” — the reason being the highly oscillatory behaviour of higher order Gegenbauer polynomials. It is actually instructive to quantify this statement, for example for the simplest, but phenomenologically very relevant case of one meson in the initial or final state, so that the convolution integral reads

$$I = \int_0^1 du \phi(u) T(u), \quad (2.7)$$

where T is the perturbative scattering amplitude.

We distinguish the following cases:

- (i) T is nonsingular for $u \in [0, 1]$;
- (ii) T has an integrable singularity at one of the endpoints;
- (iii) T contains a nonintegrable singularity at one of the endpoints.

As a typical example for case (i), consider $T(u) = \sqrt{u}$, which yields

$$\int_0^1 du \phi(u) \sqrt{u} = \sum_{n=0}^{\infty} \frac{(-1)^n 36(n+1)(n+2)}{(2n-1)(2n+1)(2n+3)(2n+5)(2n+7)} a_n.$$

This result implies a strong fall-off $\sim 1/n^3$ of the coefficients of higher Gegenbauer moments a_n : assuming $a_i \equiv 1$ for all i , already the first three terms in the sum account for 98.8% of

the full amplitude. In reality the convergence will be even better as all existing evidence points at $|a_n| \ll 1$ for $n \geq 1$.

As an example for case (ii) choose $T(u) = \ln u$, which yields

$$\int_0^1 du \phi(u) \ln(u) = -\frac{5}{6} a_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+3)} 3a_n.$$

The singularity at $u = 0$ evidently worsens the convergence of the series; again assuming $a_i \equiv 1$, the first three terms now overshoot the true result by 35%. In order to approximate the full amplitude to within 5% one now has to include nine terms, but the convergence will again be better in practice, thanks to the fall-off of a_n in n .

Case (iii) is more complicated and depends on the asymptotic behaviour of the a_n . For $T(u) = 1/u$, for instance, one obtains

$$\int_0^1 du \phi(u) \frac{1}{u} = 3 \sum_{n=0}^{\infty} (-1)^n a_n. \quad (2.8)$$

Here the amplitude is finite only if the a_n fall off sufficiently fast in n . For stronger endpoint divergences the coefficients multiplying a_n start to grow in n and for $T \sim 1/u^2$ the integral diverges, even for the asymptotic DA, which would indicate a breakdown of factorisation for that process.

The conclusion to be drawn from this discussion is that models for ϕ , based on a conformal expansion that is truncated after the first few terms, are indeed appropriate for cases (i) and (ii), but less so for case (iii). Convolutions with $T \sim 1/u$ are actually very relevant both in hard perturbative QCD reactions, e.g. $\gamma\gamma^* \rightarrow \pi$, and in B physics, e.g. $B \rightarrow \pi\pi$ [7]. Given the different weight the a_n do have in different convoluted amplitudes, and the fact that their impact is highest in convolutions of type (iii), it appears not unreasonable to base a parameterisation of ϕ not on, say, a_2 and a_4 , setting all $a_{n>4} = 0$, but rather on the “worst case scenario” of Eq. (2.8), where all a_n enter with the highest possible weight factor. This is the basic idea of our models of leading-twist DAs we shall elaborate on in the next section.

3. Models for ϕ_π and ϕ_K

There are basically two properties that any viable model for the leading-twist π DA ϕ_π must fulfill:

- (a) in the limit of large energies, $\mu^2 \rightarrow \infty$, the DA must approach the asymptotic DA $\phi_\pi(u, \mu^2 = \infty) = 6u\bar{u}$ [2];
- (b) the first inverse moment, $\int_0^1 du \phi_\pi(u)/u$, which is related to the $\pi\gamma\gamma^*$ transition form factor, must exist.

Condition (a) must evidently be fulfilled also for all other light-meson DAs; condition (b) should be fulfilled in general if QCD factorisation in B decays [7] is to make sense. Obviously any model that is based on a truncated conformal expansions fulfills both constraints, and

in addition predicts that for $u \rightarrow 0, 1$, $\phi \sim u(1-u)$ independent of the factorisation scale. We shall show that this prediction is, in general, not fulfilled for models that are not truncated at fixed order in the conformal expansion.

The starting point for our models is the relation between the first inverse moment of ϕ and the sum of all Gegenbauer moments:

$$\int_0^1 du \frac{\phi(u, \mu^2)}{3u} \equiv \Delta(\mu^2) = 1 + \sum_{n=1}^{\infty} (-1)^n a_n(\mu^2). \quad (3.1)$$

As mentioned before, one has to distinguish between mesons for which a_{odd} vanish due to G-parity, such as the π , and strange mesons, for which the odd moments induce an antisymmetric part of the DA. It is the former case we shall study first. Available experimental data for the π , as summarised in Ref. [22], point at a value of Δ around 1.1 at the scale $\mu \approx 1.2 \text{ GeV}$, which implies that the infinite sum be convergent. Hence, even at the comparatively low scale 1.2 GeV the a_{2n} must fall off fast enough in n . Assuming an asymptotic equal-sign behaviour for large n , a power-like fall-off as $a_{2n} \sim 1/n^p$ with p slightly larger than 1 is one of the slowest possible fall-offs. It turns out that DAs defined by a power-like fall-off of the Gegenbauer moments can actually be summed explicitly: using the generating function of Gegenbauer polynomials,

$$f(\xi, t) = \frac{1}{(1 - 2\xi t + t^2)^{3/2}} = \sum_{n=0}^{\infty} C_n^{3/2}(\xi) t^n,$$

the DA with moments

$$a_n = \frac{1}{(n/b + 1)^a}, \quad \text{for } n \text{ even,}$$

is given by

$$\tilde{\phi}_{a,b}^+(u) = \frac{3u\bar{u}}{\Gamma(a)} \int_0^1 dt (-\ln t)^{a-1} \left(f(2u-1, t^{1/b}) + f(2u-1, -t^{1/b}) \right).$$

In the same way one obtains a DA with alternating-sign behaviour of the Gegenbauer moments,

$$a_n = \frac{(-1)^{n/2}}{(n/b + 1)^a}, \quad \text{for } n \text{ even,}$$

as

$$\tilde{\phi}_{a,b}^-(u) = \frac{3u\bar{u}}{\Gamma(a)} \int_0^1 dt (-\ln t)^{a-1} \left(f(2u-1, it^{1/b}) + f(2u-1, -it^{1/b}) \right).$$

The corresponding values of Δ are $\Delta_{a,b}^+ = (b/2)^a \zeta(a, b/2)$ and $\Delta_{a,b}^- = (b/4)^a (\zeta(a, b/4) - \zeta(a, 1/2 + b/4))$, where $\zeta(a, s) = \sum_{k=0}^{\infty} 1/(k+s)^a$ is the Hurwitz ζ function. In order to obtain models for arbitrary values of Δ , we split off the asymptotic DA and write

$$\phi_{a,b}^{\pm}(\Delta) = 6u\bar{u} + \frac{\Delta - 1}{\Delta_{a,b}^{\pm} - 1} \left(\tilde{\phi}_{a,b}^{\pm}(u) - 6u\bar{u} \right), \quad \text{valid for } a \geq 1 \text{ and } b > 0. \quad (3.2)$$

Is there any reason why the a_n should fall off as inverse powers in n — other than that the corresponding DAs can be expressed in closed form via Eq. (3.2)? The answer to this

question relies on the behaviour of $a_n(\mu^2)$ under a change of scale. Evidently the models are defined at one particular scale, for instance the hadronic scale $\mu \approx 1.2 \text{ GeV}$. At larger scales Q^2 , the a_n change according to Eq. (2.4). For large n , we have

$$\gamma_0^{(n)} \stackrel{n \rightarrow \infty}{\approx} 8C_F \ln n + O(1)$$

and

$$a_n(Q^2) \stackrel{n \rightarrow \infty}{\approx} \frac{1}{n^{4C_F/\beta_0 \ln(1/L)}} a_n(\mu^2)$$

with $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$. That is: perturbative leading-order scaling induces a power-like fall-off of the a_n , at least for large n . We take this as indication that such a behaviour is indeed intrinsic to QCD. One more consequence of scaling is that for $Q^2 \rightarrow \infty$, i.e. $L \rightarrow 0$, the suppression of higher a_n is power-like with a power that approaches infinity, so that

$$\tilde{\phi}^\pm(Q^2 \rightarrow \infty) = \tilde{\phi}^\pm(a \rightarrow \infty) = 6u(1-u).$$

Hence $\phi_{a,b}^\pm$ as defined in (3.2) approaches the asymptotic DA in this limit, which implies that condition (a) follows from (b).

Eq. (3.2) implies that the asymptotic DA is recovered for $\Delta = 1$, and also from $\phi_{a,b}^+$ in the limit $a \rightarrow 1$. The models are valid only for $a \geq 1$, as otherwise $\Delta_{a,b}^+$ diverges, or, equivalently, $\phi_{a,b}^\pm$ does not vanish at the endpoints $u = 0, 1$. In Fig. 1 we plot several examples of $\phi_{a,b}^\pm$ for a fixed value of Δ ; it is evident that the two models $\phi_{a,b}^+$ and $\phi_{a,b}^-$ have a rather dissimilar functional dependence on u ; in particular $\phi_{a,b}^-$ turns out to be nonanalytic at $u = 1/2$ for $a \leq 3$. The dependence of $\phi_{a,b}^\pm$ on b is illustrated in Fig. 2. In Fig. 3 we show the possible values of the lowest Gegenbauer moments $a_{2,4}$ that can be obtained for fixed values of Δ and b , but different a . For $\phi_{a,b}^+$, one always has $a_2 < \Delta - 1$, for $\phi_{a,b}^-$, $a_2 > \Delta - 1$; in the limit $a \rightarrow \infty$ both branches meet and one obtains the truncated NLO conformal expansion with $a_2 = \Delta - 1$ and $a_4 = 0$.

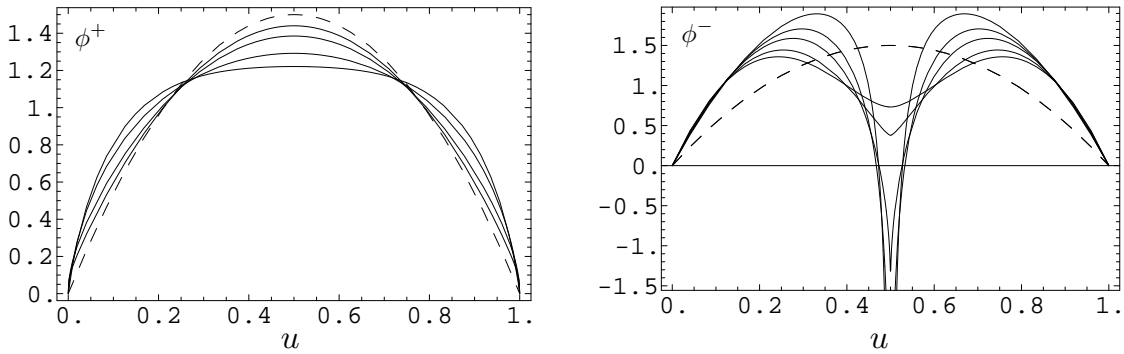


Figure 1: Left: Examples for model DAs $\phi_{a,b}^+$ as functions of u , for $a = 1.5, 2, 3, 4$, $b = 2$ and $\Delta = 1.2$ (solid curves), as compared to the asymptotic DA (dashed curve). For $a \rightarrow 1$, $\phi_{a,b}^+$ approaches the asymptotic DA. Right: the same for $\phi_{a,b}^-$. Note that for $a \leq 3$, $\phi_{a,b}^-$ is nonanalytic at $u = 1/2$ and displays a pronounced “spike”.

Before discussing constraints on the model parameters Δ , a and b , let us shortly comment on the behaviour of the DAs near the endpoints $u = 0, 1$. In many applications of

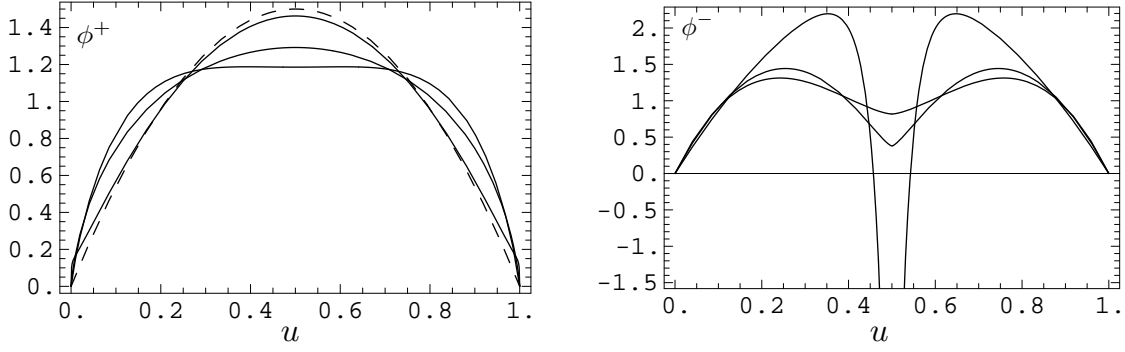


Figure 2: Left: Examples for model DAs $\phi_{3,b}^+$ as functions of u , for $b = 0.2, 2, 20$ and $\Delta = 1.2$ (solid curves), as compared to the asymptotic DA (dashed curve). For $b \rightarrow \infty$, $\phi_{a,b}^+$ approaches the asymptotic DA, as $\Delta_{a,\infty}^+ \rightarrow \infty$. Right: the same for $\phi_{3,b}^-$.

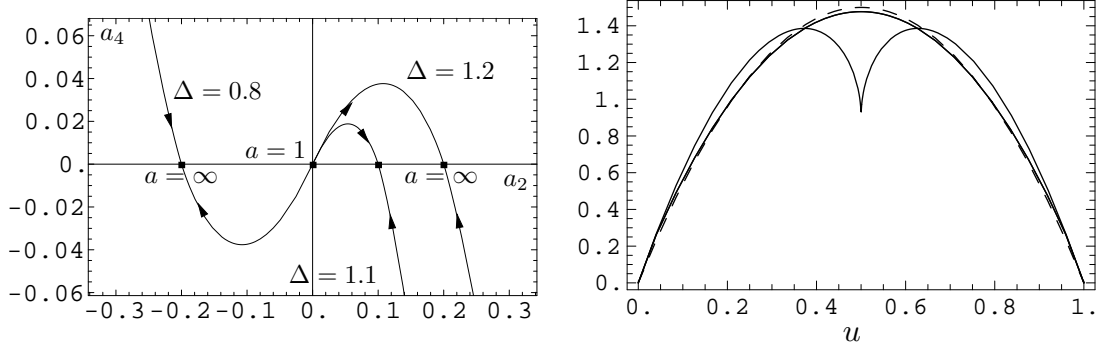


Figure 3: Values of a_2 and a_4 for different parameters Δ and a of $\phi_{a,2}^\pm$. The curves in the upper right and lower left quadrant correspond to $\phi_{a,2}^+$, those in the lower right and upper left one to $\phi_{a,2}^-$.

Figure 4: Comparison of $\phi_{2,2}^\pm(u)$ (solid curves) with the asymptotic DA (dashed curve) for $\Delta = 1.04$. The “spike” at $u = 1/2$ characteristic for ϕ^- causes these models to significantly deviate from the asymptotic DA even for Δ close to 1.

DAs it is assumed – implicitly or explicitly – that $\phi \sim u(1-u)$ near the endpoints. Although this is evidently the case for any model based on a truncated conformal expansion, it does not apply to our models, at least not for all values of a . A closer inspection shows that $\phi^+ \sim \sqrt{u(1-u)}$ for $a = 2$ and $\sim u(1-u) \ln(u(1-u))$ for $a = 3$. It is only for $a > 3$ that the DAs behave in the “canonical” linear way. Are there any rigorous arguments why the DAs should behave linearly near the endpoints, even for low hadronic scales? As far as we could trace the origin of the argument in favour of linear behaviour, it was stated first in Ref. [9] and relies on the fact that QCD sum rules for the moments of the DA exhibit the following behaviour:

$$\langle \xi^n \rangle = \int_0^1 \xi^n \phi(u) \, n \xrightarrow{\infty} \frac{1}{n^2}.$$

Two comments are in order here. First, an exact calculation for arbitrary ϕ shows that

$$\langle \xi^n \rangle = \frac{1}{4n^2} (\phi'(0) - \phi'(1)) + O(n^{-3}),$$

so that the correct statement is that $\langle \xi^n \rangle \sim 1/n^2$ implies that $\phi'(u)$ exists at the endpoints – which in turn indeed implies a linear behaviour, since $\phi(0) = 0 = \phi(1)$ as long as $\Delta < \infty$. Second, the conclusion of the authors of [9] is based on results obtained from the leading-order perturbative contributions to QCD sum rules. NLO expressions have been obtained in [16, 17] and yield $\phi \sim u(1-u) \ln^2(u/(1-u))$, which upsets the linear approach to the endpoints and is equivalent to $a_n \sim 1/n^3$. The large- n behaviour of nonperturbative terms cannot be obtained from these sum rules,² but we see no reason why it should not follow the $a_n \sim 1/n^3$ behaviour of NLO perturbation theory or even introduce $1/n^2$ scaling. We hence conclude that the standard assumption of $\phi \sim u(1-u)$ is not rigorously justified at hadronic scales and that all corresponding conclusions, in particular the scaling behaviour of the $B \rightarrow$ light meson form factors at zero momentum transfer, $f(0) \sim 1/m_b^{3/2}$, first derived in [24], should be taken *cum grano salis*. We also would like to add one more remark about the expectation that for small $\Delta - 1$ the DA should be “very close” to the asymptotic DA. In Fig. 4 we show that this statement is indeed true for ϕ^+ models, but not for ϕ^- , which are characterised by their nonanalytic behaviour at $u = 1/2$. Of course the asymptotic DA is recovered for $\Delta - 1 \rightarrow 0$, but not in an analytic way.

Let us now discuss the experimental restrictions on Δ , a and b for the π . First, we fix the reference scale as $\mu = 1.2 \text{ GeV}$. As the data are too scarce to constrain all parameters in a meaningful way, we fix $b \equiv 2$. As for Δ , we require it to be larger or equal 1. This is the case if $a_2 \geq 0$, which is

a	2	3	4	5	6	∞
Δ_{max}^+	2.04	1.58	1.43	1.36	1.33	1.27
Δ_{max}^-	1.04	1.11	1.16	1.19	1.22	1.27

Table 1: Upper bound on Δ^\pm , for various values of a , as implied by $\phi_\pi(1/2) > 0.9$. The reference scale is $\mu \approx 1.2 \text{ GeV}$.

indeed the common overall conclusion of all determinations available in the literature, as compiled in Ref. [22]. An upper bound on Δ can be inferred from experimental data or from requiring $a_2 \leq 0.2$, which is again the maximum value allowed by most analyses.³ For ϕ^+ the restriction on a_2 implies $\Delta \leq 1.2$, which coincides with the allowed range of Δ found in [19], whereas for ϕ^- the resulting range of Δ is smaller. One more constraint on ϕ^\pm is the allowed range of $\phi_\pi(1/2)$, which follows from light-cone sum rules for the πNN coupling [15]:

$$0.9 \leq \phi_\pi(1/2, 1 \text{ GeV}) \leq 1.5.$$

For $\Delta > 1$, $\phi(1/2)$ is always smaller than 1.5, so it is only the lower bound that is relevant. In Tab. 1 we list the corresponding maximum Δ for various a . Evidently the constraints on

²The reason being that a_n for large n are intrinsically nonlocal quantities, which cannot be obtained from a local operator product expansion (OPE). The ineligibility of the local OPE manifests itself as contributions to a_n that scale as *positive* powers in n , which is incompatible with a finite value of Δ .

³With the notable exception of Chernyak and Zhitnitsky [9], whose results are however by now generally considered to be excluded by experiment.

Δ^+ are weaker than the ones discussed before, but for ϕ^- the minimum value of $\phi_\pi(1/2)$ poses a nontrivial constraint on Δ .

It is possible to further refine the constraints for the π as it was done in e.g. [19, 21]. In this paper, however, we are not so much interested in the π , but rather in the gross characteristics of the DAs, which are likely to be valid also for other pseudoscalar and vector mesons. We hence refrain from pursuing that line of investigation, but summarise the main constraints on the symmetric parts of our model DAs, which are likely to be valid also for other mesons:

- $1 \leq \Delta \leq 1.2$ for $0 \leq a_2 \leq 0.2$ for $\phi_{a,2}^+$: this is based on the observation [9] that DAs of mesons with higher mass tend to become narrower;
- $1 \leq \text{Min}(1.2, \Delta_{\text{max}}^-)$ for $\phi_{a,2}^-$, with Δ_{max}^- given in Tab. 1;
- $b = 2$, lacking further data.

Let us now turn to ϕ_K , the twist-2 DA of the K meson. It differs from ϕ_π by the fact that now also odd Gegenbauer moments contribute. Models for the antisymmetric part of the DA can be constructed in a similar way as before as⁴

$$\begin{aligned}\tilde{\psi}_c^+(u) &= \frac{3u\bar{u}}{\Gamma(c)} \int_0^1 dt (-\ln t)^{c-1} \left(f(2u-1, \sqrt{t}) - f(2u-1, -\sqrt{t}) \right), \\ \tilde{\psi}_c^-(u) &= \frac{3u\bar{u}}{i\Gamma(c)} \int_0^1 dt (-\ln t)^{c-1} \left(f(2u-1, i\sqrt{t}) - f(2u-1, -i\sqrt{t}) \right).\end{aligned}\quad (3.3)$$

The models are characterised by c and yield $a_1 = (2/3)^c$. Models for the antisymmetric part of ϕ with arbitrary a_1 can then be defined as

$$\psi_c^\pm = a_1 (3/2)^c \tilde{\psi}_c^\pm(u). \quad (3.4)$$

Examples for such models are shown in Fig. 5.

Numerical values for a_1^K have been discussed in [6]; we quote $a_1^K(1.2 \text{ GeV}) \approx 0.15$. For ϕ_K , the total Δ_{tot} can then be written as

$$\Delta^{\text{tot},\pm} = \Delta + \Delta^{\text{asym},\pm},$$

where $\Delta = \sum a_{\text{even}}$ is the contribution of the symmetric part of the DA to Δ^{tot} and $\Delta^{\text{asym},\pm}$ is defined as

$$\begin{aligned}\Delta^{\text{asym},+} &= \int_0^1 du \frac{\psi_c^+(u)}{3u} = -a_1 (3/2)^c \zeta(c, 3/2), \\ \Delta^{\text{asym},-} &= \int_0^1 du \frac{\psi_c^-(u)}{3u} = -a_1 (3/4)^c \{ \zeta(c, 3/4) - \zeta(c, 5/4) \}.\end{aligned}$$

⁴In complete analogy to the symmetric part of the DA we could introduce one more parameter d that would correspond to b . In view of the near complete absence of any information on antisymmetric DAs, we refrain from writing down the corresponding formulas and set $d \equiv 2$ from the very beginning.

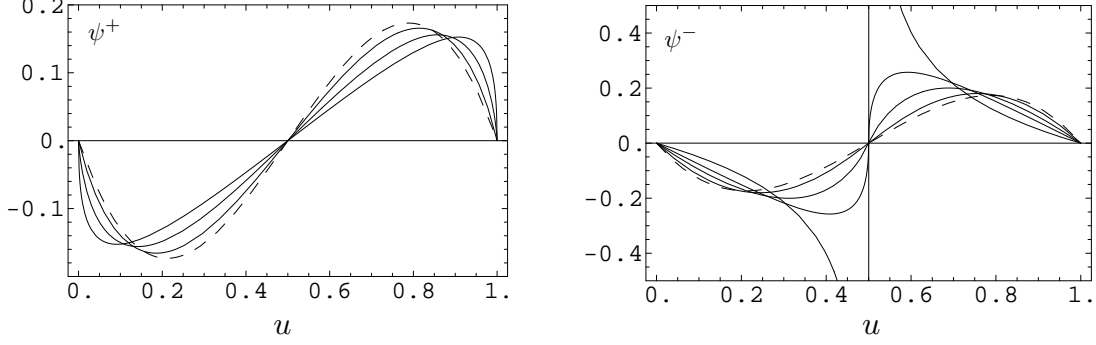


Figure 5: Models for the antisymmetric contributions to the twist-2 DA for $a_1 = 0.15$. Left: ψ_c^+ as function of u for $c \in \{2, 3, 5\}$ (solid curves), dashed curve: ψ_∞^+ . Right: ψ_c^- as function of u for $c \in \{1, 2, 3, 5\}$ (solid curves), dashed curve: ψ_∞^- . Like the symmetric models ϕ_a^- , ψ_c^- is nonanalytic at $u = 1/2$ for $c \leq 3$.

4. Results for $f_+^{B \rightarrow \pi}(0)$

One important application of factorisation in B physics is the calculation of the weak decay form factor $f_+^{B \rightarrow \pi}$ from QCD sum rules on the light-cone. It is beyond the scope of this paper to review the method of QCD sum rules on the light-cone (LCSRs), for which we refer to Ref. [4]. Instead, we would like to stress that LCSRs offer a means to calculate nonperturbative hadronic quantities like form factors within a controlled approximation, which relies on the factorisation of an unphysical correlation function, whose imaginary part is related to the hadronic quantity in question.

The $B \rightarrow \pi$ form factors are defined as

$$\langle \pi(p) | V_\mu | B(p_B) \rangle = \left[(p + p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right] f_+^{B \rightarrow \pi}(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu f_0^{B \rightarrow \pi}(q^2), \quad (4.1)$$

with $q = p_B - p$. Within the LCSR method $f_{+,0}^{B \rightarrow \pi}$ can be related to a correlation function

$$\begin{aligned} \Pi_\mu(q, p_B) &= i \int d^4x e^{iq \cdot x} \langle \pi(p) | T V_\mu(x) j_B^\dagger(0) | 0 \rangle \\ &= \Pi_+(q^2, p_B^2)(p + p_B)_\mu + \Pi_-(q^2, p_B^2)q_\mu, \end{aligned} \quad (4.2)$$

where $j_B = m_b \bar{d} i \gamma_5 b$ is the interpolating field for the B meson. For unphysical $p_B^2 \ll m_b^2$, Π_\pm can be expanded around the light-cone as

$$\Pi_\pm^{\text{LC}}(q^2, p_B^2) = \sum_n \int_0^1 du T_\pm^{(n)}(u, q^2, p_B^2, \mu) \phi^{(n)}(u, \mu), \quad (4.3)$$

where the sum runs over contributions of increasing twist and the term in $n = 2$ is just the leading-twist contribution. $T_\pm^{(2,3)}$ are known to $O(\alpha_s)$ [6], whereas $T_\pm^{(4)}$ is known at tree-level [5]. We have extended the calculation performed in [6] to include contributions up to a_{10} . The LCSR for the form factor f_+ depends on the spectral density ρ_+^{LC} of Π_+^{LC} in p_B^2 and reads

$$e^{-m_B^2/M^2} m_B^2 f_B f_+^{B \rightarrow \pi}(q^2) = \int_{m_b^2}^{s_0} ds e^{-s/M^2} \rho_+^{\text{LC}}(s, q^2), \quad (4.4)$$

where M^2 , the so-called Borel parameter, and s_0 , the continuum-threshold, are sum rule specific parameters. Using the central values of these parameters as obtained in [6], $M^2 = 9.2 \text{ GeV}^2$ and $s_0 = 33.9 \text{ GeV}^2$, and the model twist-2 DAs $\phi_{a,2}^\pm$ we obtain the values of $f_+^{B \rightarrow \pi}(0)$ shown in Fig. 6. The dependence of $f_+^{B \rightarrow \pi}(0)$ on Δ , with a fixed, is linear, as the

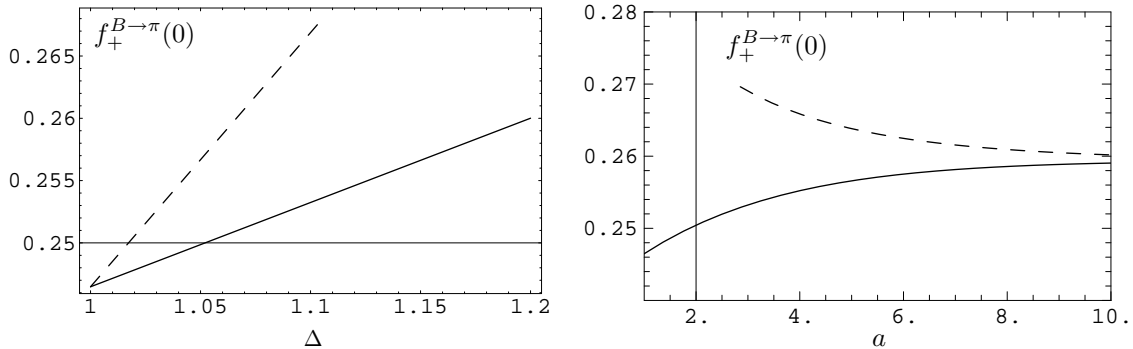


Figure 6: $f_+^{B \rightarrow \pi}(0)$ calculated from LCSRs, using the twist-2 DAs $\phi_{a,2}^+$ (solid lines) and $\phi_{a,2}^-$ (dashed lines). Left: $f_+^{B \rightarrow \pi}(0)$ as function of Δ for $a = 3$ fixed; right: $f_+^{B \rightarrow \pi}(0)$ as function of a for $\Delta = 1.1$ fixed. The endpoints of the dashed curves are set by the constraint $\phi(1/2) \geq 0.9$.

values of the individual $a_{n \geq 2}$ are just rescaled by a common factor. If, on the other hand, Δ is kept fixed and a is being varied, higher a_n are increasingly suppressed with increasing a , so that for $a \rightarrow \infty$ the form factor obtained using ϕ^+ approaches that obtained using ϕ^- , as is clearly visible in Fig. 6.

Figure 6 shows that the theoretical uncertainty induced by Δ and a is about $\pm 5\%$. This has to be compared with the final value of $f_+^{B \rightarrow \pi}(0)$ stated in Ref. [6]: 0.258 ± 0.031 , i.e. a 12% uncertainty, which includes also the variation of other input parameters of the LCSR.

5. $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ in QCD Factorisation

Another important application of leading-twist DAs are nonleptonic B decays treated in QCD factorisation [7]. Recent experimental data point at a failure of QCD factorisation to explain the observed branching ratios and CP asymmetries in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$, which has prompted a number of authors to explain this effect by new physics, e.g. Ref. [25], but has also motivated other authors to investigate the impact of nonfactorisable corrections to QCD factorisation, which are suppressed by inverse powers of m_b [26]. In this section we investigate the effect of nonstandard DAs on the predictions of QCD factorisation. The factorisation formulas depend on twist-2 DAs of the π and K , but also on the form factor $f_+^{B \rightarrow \pi}(0)$, for which we use the results from LCSRs, cf. Sec. 4.

Let us first study the time-dependent CP-asymmetry in $B \rightarrow \pi^+\pi^-$, which is defined as

$$A_{CP}^{\pi\pi} = \frac{\Gamma(\bar{B}_0 \rightarrow \pi^+\pi^-) - \Gamma(B_0 \rightarrow \pi^+\pi^-)}{\Gamma(\bar{B}_0 \rightarrow \pi^+\pi^-) + \Gamma(B_0 \rightarrow \pi^+\pi^-)} = S_{\pi\pi} \sin \Delta m t + C_{\pi\pi} \cos \Delta m t, \quad (5.1)$$

where our interest is in the mixing-induced asymmetry $S_{\pi\pi}$ which depends on the unitarity triangle (UT) angles β and γ via

$$S_{\pi\pi} = \frac{2\text{Im}\lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2}, \quad \lambda_{\pi\pi} = e^{-2i\beta} \frac{e^{-i\gamma} + P/T}{e^{i\gamma} + P/T}. \quad (5.2)$$

In the limit where the penguin-to-tree ratio P/T is zero this reduces to $S_{\pi\pi} = \sin 2\alpha$. Although P/T is highly suppressed, it is not negligible and can be expressed, in QCD factorisation, in terms of the CKM parameters and the Gegenbauer moments a_n . Neglecting the small contributions from weak annihilation terms [7], we find that P/T is given by a ratio of polynomials in a_n with complex coefficients. For the asymptotic DA, for instance, one has

$$\frac{P}{T} = -\frac{1}{R_b} \frac{(-1.36 - 0.31i) - (1.46 + 0.37i)a_0 + (0.22 + 0.01i)a_0^2}{(13.0 - 0.08i) + (15.5 + 0.03i)a_0 - (0.56 - 0.15i)a_0^2}, \quad (5.3)$$

where $R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 0.40$ [27].

The variation of $S_{\pi\pi}$ in terms of a and Δ is shown in Fig. 7. For $1 \leq \Delta \leq 1.2$ as suggested in Sec. 3, the variation is of order 1% and becomes more significant only for unrealistically large values of Δ ; the convergence for $\Delta = 1$ corresponds to the asymptotic wave function. The current experimental results [28] are

$$S_{\pi\pi} = -0.30 \pm 0.17 \pm 0.03 \quad (\text{BaBar}), \quad S_{\pi\pi} = -1.00 \pm 0.21 \pm 0.07 \quad (\text{Belle}), \quad (5.4)$$

which could only be accommodated using very extreme values of Δ . For example, taking a model with same-sign fall-off, to reproduce the BaBar result with $a = 2$ would require a value of $\Delta > 10$ to be within the 1σ band and $\Delta = 20$ to approach the central value. Even for higher values of a , a minimum $\Delta \approx 16$ is needed to approach $S_{\pi\pi} = -0.3$, and would produce Gegenbauer moments that are significantly outside the known constraints, for example with $a = 6$ and $\Delta = 16$, we find $a_2(2.2 \text{ GeV}) = 7.7$ and $a_4(2.2 \text{ GeV}) = 0.5$. Obtaining a value of $S_{\pi\pi} = -1.0$ is not possible for values of $\Delta \geq 1$, which is required to keep a_2 positive.

The effect of the DA model on the branching ratios of $B \rightarrow \pi^+\pi^-$ is significantly more pronounced than for the CP-asymmetry. The central value of the branching ratio for the asymptotic DA is

$$\text{BR}(B \rightarrow \pi^+\pi^-) = 5.5 \times 10^{-6} |0.25e^{i \cdot 15^\circ} + e^{-i\gamma}|^2, \quad (5.5)$$

where $\gamma = 60^\circ \pm 7^\circ$ [27], and the explicit dependence of the branching ratio on the Gegenbauer moments is again a polynomial in a_n :

$$\begin{aligned} \text{BR}(B \rightarrow \pi^+\pi^-) \approx & 1.50 \times 10^{-6} |1 + (1.17 - 0.01i)a_0 - (0.06 + 0.01i)a_0^2 \\ & + (1.16 - 0.01i)a_2 - (0.02 + 0.01i)a_2^2 - (0.08 + 0.02i)a_0a_2 + \dots|^2, \end{aligned} \quad (5.6)$$

where the terms of first order in a_n come from nonspectator interaction and those of second order from the hard-gluon exchange. Figure 7 shows the variation of the branching ratio for the two model DAs, which has to be compared to the current experimental world

average $\text{BR}(B \rightarrow \pi^+\pi^-) = (4.6 \pm 0.4) \times 10^{-6}$ [29]. We see that large increases from the asymptotic value are possible by increasing Δ , with values within the physical range, with a considerable effect of around 30% possible with the alternating-sign fall-off model. On the other hand, the effect of a nonasymptotic DA is to *increase* the branching ratio, and hence leads the result of QCD factorisation even further away from the experimental result.

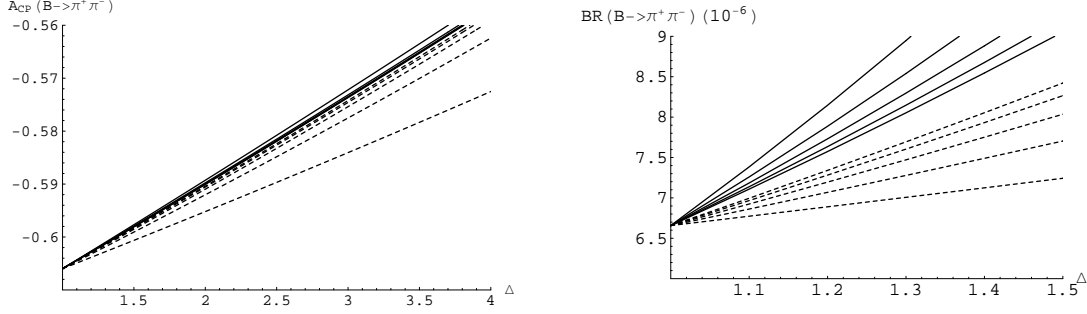


Figure 7: Left: $A_{CP}(\pi\pi)$ as a function of Δ for $a = 2, 3 \dots 6$, shown for models DAs with same-sign fall-off (dashed lines) and alternating sign fall-off (solid lines). Right: the same for $\text{BR}(B \rightarrow \pi^+\pi^-)$. Both sets of curves converge for increasing values of a .

The situation for $B \rightarrow \pi K$ decays is complicated by the presence of the K DA ϕ_K , for which very little is known about the Gegenbauer moments. As a result of this, we use the same moments for ϕ_K as in ϕ_π with the addition of the parameter a_1^K , for which we use the value of $a_1^K(1.2 \text{ GeV}) = 0.15$, as discussed in Sec. 3. Concentrating on the decay $\bar{B}_0 \rightarrow \pi^+ K^-$ and its CP-conjugate, we consider first the direct CP asymmetry, recently reported in Ref. [30] as

$$\begin{aligned} A_{CP}^{\pi K} &= -0.133 \pm 0.030 \pm 0.009 \quad (\text{BaBar}), \\ A_{CP}^{\pi K} &= -0.101 \pm 0.025 \pm 0.005 \quad (\text{Belle}). \end{aligned} \quad (5.7)$$

In the framework of QCD factorisation, A_{CP} can be written in terms of real, strong interaction parameters (derived from the factorisation coefficients) and pure CKM variables. We can neglect the annihilation contributions, but the electroweak penguins play a crucial role and must therefore be included, which yields

$$A_{CP}^{\pi K} = \frac{\tan^2 \theta_c R_b (\sin \gamma) r(\pi K)}{1 + \tan^2 \theta_c R_b (\cos \gamma) r'(\pi K)}, \quad (5.8)$$

where θ_c is the Cabibbo angle and $r, r'(\pi K)$ contain the QCD effects which depend on the DA model parameters Δ and a .

Figure 8 shows the dependence of the direct CP-asymmetry in $\bar{B}_0 \rightarrow \pi^+ K^-$ on Δ and the fall-off parameter a . The first point to note is that A_{CP} differs in sign from the experimental value (5.7), which is in agreement with the findings of [7]. The precise value of A_{CP} does depend on Δ , but it is impossible, even for extreme DAs, to obtain the experimentally observed negative sign. This is also emphasised by plotting the dependence of the asymmetry on the UT angle γ , as expressed in (5.8), which is also shown Fig. 8.

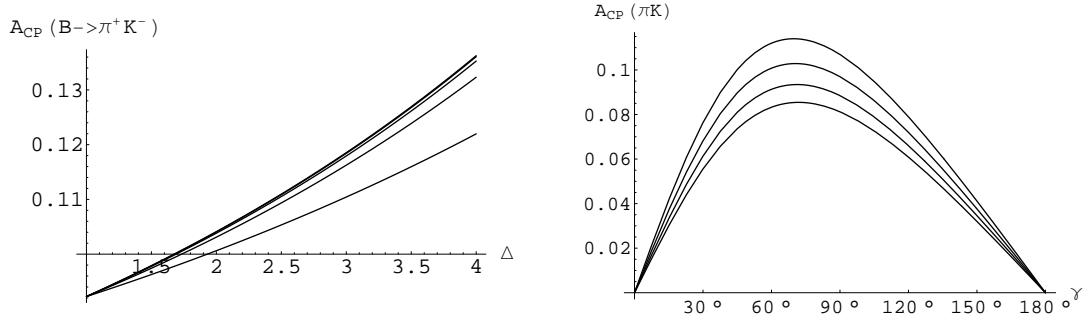


Figure 8: Left: the variation of $A_{CP}^{\pi K}$ as a function of Δ for $a = 2, 3 \dots 6$, for same-sign fall-off. Right: the dependence of $A_{CP}^{\pi K}$ on the UT angle γ for curves at $a = 2$, $\Delta = 1$ (lowest curve) to $\Delta = 4$.

There is a 10% increase in the value of $A_{CP}(\pi K)$ between the asymptotic form with $\Delta = 1$ and $\Delta = 2$, more significant changes only occurring outside the physical range of Δ .

As in the $B \rightarrow \pi^+ \pi^-$ case, the effect of our model DAs on the branching ratio of $\bar{B}_0 \rightarrow \pi^+ K^-$ is much more pronounced than for the CP-asymmetry and can exhibit up to 30% change from the asymptotic value within the physical ranges of a and Δ . The branching ratio is given as

$$\text{BR}(B \rightarrow \pi K) \propto (F_0^{B \rightarrow \pi}(m_K^2))^2 |V_{cb} V_{cs}^*|^2 \left[\epsilon_{KM} e^{-i\gamma} c(\pi K) + c'(\pi K) \right]^2, \quad (5.9)$$

with $\epsilon_{KM} = \tan \theta_c^2 R_b$ and $c, c'(\pi K)$ are derived from factorisation coefficients, containing all the dependence on hadronic parameters and the DAs $\phi_{\pi, K}$. Using the asymptotic DA we find $\text{BR}(B \rightarrow \pi K) = 13.58 \times 10^{-6}$, and the variation from this when higher-order Gegenbauer moments are included is shown in Fig. 9. The experimental world average,

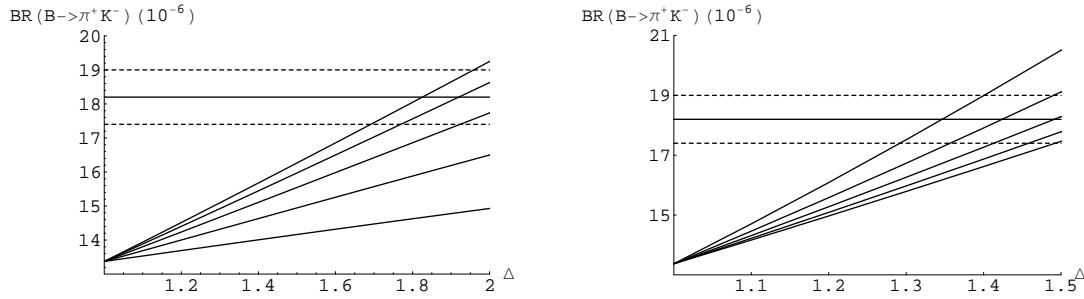


Figure 9: $\text{BR}(B \rightarrow \pi^+ K^-)$ as a function of Δ for $a = 2, 3 \dots 6$, for models DAs with same-sign fall-off (left plot) and alternating sign fall-off (right plot). The experimental average is also marked for comparison.

also shown in Fig. 9, is $(18.2 \pm 0.8) \times 10^{-6}$ [29]. There can be significant changes to the asymptotic value, especially in the model with alternating sign fall-off. The experimental average is shown for comparison and can be accommodated with $\Delta \approx 1.8$ for the same sign fall-off, or $\Delta \approx 1.4$ with alternating sign fall-off.

In Fig. 10 we also show the dependence of the direct CP-asymmetry in $B \rightarrow \pi K$ on a_1 , the Gegenbauer moment that parameterises the antisymmetric part of the K DA, whose actual value is around 0.15. It is evident that one can shift A_{CP} into the “right” direction by decreasing a_1 , but again extreme values of a_1 would be needed to obtain a negative CP-asymmetry.

Our conclusion from this investigation is that the discrepancy between the experimental values of BRs and CP-asymmetries and their predicted values in QCD factorisation can not be attributed to the uncertainties in the leading-twist DAs: for $B \rightarrow \pi^+\pi^-$ it is impossible to reproduce the data, whereas for the direct CP-asymmetry in $B \rightarrow \pi K$ highly unrealistic values of Δ and a would be needed in order to reconcile the theory predictions with experimental data.

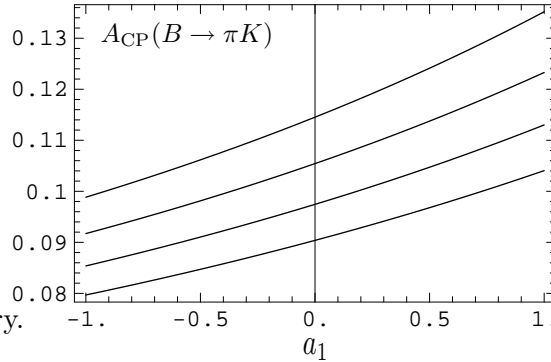


Figure 10: The direct CP-asymmetry in $\bar{B}_0 \rightarrow \pi^+ K^-$ as function of a_1 for $a = 3$ and $\Delta \in \{1, 2, 3, 4\}$ (from bottom to top). $\gamma = 60^\circ$.

6. Summary and Conclusions

We have presented models for the symmetric part of leading-twist light-cone distribution amplitudes (DAs) of light mesons which depend on three parameters. Two of these parameters control the fall-off behaviour of the Gegenbauer moments a_n in n , whereas the third one, Δ , is given by the first inverse moment of the DA and parameterises the maximum possible impact of higher Gegenbauer moments on the actual physical amplitude described in factorisation. We have also developed similar models for the antisymmetric part of the DA, which is relevant for K and K^* mesons; these models are normalised to a_1 , the first Gegenbauer moment. For the π DA, for which experimental data exist, we have formulated constraints on the model parameters which are likely to be valid also for other meson DAs. We have argued that these models are better suited to estimate the true hadronic uncertainty of processes calculated in factorisation than the standard truncated conformal expansion and have studied these uncertainties for two quantities, the $B \rightarrow \pi$ weak decay form factor $f_+^{B \rightarrow \pi}(0)$ and the CP asymmetry in $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$. For the former, the theoretical uncertainty induced by the model-dependence of the DA is smaller than that due to other parameters and approximations. For nonleptonic decays calculated in QCD factorisation we find that the branching ratios are more sensitive to the precise values of the model parameters than the CP-asymmetries. In both decays channels it is however impossible to explain the experimental data by nonstandard DAs, which indicates the presence of nonnegligible nonfactorisable contributions — a conclusion that agrees with the findings of other authors, e.g. Refs. [26, 33].

Our models should prove particularly useful for describing DAs of mesons other than the π which are also symmetric by virtue of G-parity, but for which no experimental or other reliable theoretical information is available — in particular the ρ , ω and ϕ . For

these particles, we argue that existing theoretical indications from local QCD sum rules [9, 17, 16] point at the DAs being narrower than that of the π , which implies the allowed values of Δ being smaller than 1.2 at the scale 1.2 GeV. The same results also imply a_2 to be positive, which entails $\Delta \geq 1$. For the parameter a , which controls the fall-off of the a_n in n , we have found that the perturbative contributions to QCD sum rules indicate it to be 3 [11], but that also smaller values of a are not excluded unless one can rigorously prove that the leading-twist DAs must behave as $\sim u(1-u)$ near the endpoints $u = 0, 1$, at *all scales*, which would imply $a \geq 4$. One more relevant constraint for the models $\phi_{a,b}^-$ with an alternating-sign fall-off of the Gegenbauer moments comes from the requirement $\phi_\pi(1/2) > 0.9$ at the scale $\mu = 1$ GeV. A positive value of this quantity is also required by QCD sum rules on the light-cone for the couplings $g_{DD^*\pi}$ [31] and $g_{DD^*\rho}$ [32], and hence also very likely to be the case for other mesons. But even without these constraints being taken literally, our models provide a way to test the impact of nonasymptotic DAs on physical amplitudes without recourse to conformal expansion.

Acknowledgments

A.N.T. gratefully acknowledges receipt of a UK PPARC studentship.

References

- [1] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079].
- [2] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25** (1977) 510 [Pisma Zh. Eksp. Teor. Fiz. **25** (1977) 544];
Sov. J. Nucl. Phys. **31** (1980) 544 [Yad. Fiz. **31** (1980) 1053];
A.V. Efremov and A.V. Radyushkin, Phys. Lett. B **94** (1980) 245; Theor. Math. Phys. **42** (1980) 97 [Teor. Mat. Fiz. **42** (1980) 147];
G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87** (1979) 359; Phys. Rev. D **22** (1980) 2157;
V.L. Chernyak, A.R. Zhitnitsky and V.G. Serbo, JETP Lett. **26** (1977) 594 [Pisma Zh. Eksp. Teor. Fiz. **26** (1977) 760]; Sov. J. Nucl. Phys. **31** (1980) 552 [Yad. Fiz. **31** (1980) 1069].
- [3] S.J. Brodsky and G.P. Lepage, in: *Perturbative Quantum Chromodynamics*, ed. by A.H. Mueller, p. 93, World Scientific (Singapore) 1989.
- [4] P. Colangelo and A. Khodjamirian, hep-ph/0010175;
A. Khodjamirian, hep-ph/0108205.
- [5] V.M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C **60** (1993) 349 [hep-ph/9305348];
P. Ball and V. M. Braun, Phys. Rev. D **55** (1997) 5561 [arXiv:hep-ph/9701238];
A. Khodjamirian *et al.*, Phys. Lett. B **410** (1997) 275 [hep-ph/9706303];
E. Bagan, P. Ball and V.M. Braun, Phys. Lett. B **417** (1998) 154 [hep-ph/9709243];
P. Ball, JHEP **9809** (1998) 005 [hep-ph/9802394];
P. Ball and V. M. Braun, Phys. Rev. D **58** (1998) 094016 [arXiv:hep-ph/9805422];
A. Khodjamirian *et al.*, Phys. Rev. D **62** 114002 (2000) [hep-ph/0001297].
- [6] P. Ball and R. Zwicky, JHEP **0110** (2001) 019 [arXiv:hep-ph/0110115];
P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005) [arXiv:hep-ph/0406232].

- [7] M. Beneke *et al.*, Phys. Rev. Lett. **83** (1999) 1914 [arXiv:hep-ph/9905312];
M. Beneke *et al.*, Nucl. Phys. B **606** (2001) 245 [arXiv:hep-ph/0104110];
M. Beneke and M. Neubert, Nucl. Phys. B **675** (2003) 333 [arXiv:hep-ph/0308039].
- [8] C. W. Bauer *et al.*, Phys. Rev. D **63** (2001) 114020 [arXiv:hep-ph/0011336].
- [9] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. **112** (1984) 173.
- [10] V. M. Braun and I. E. Filyanov, Z. Phys. C **48** (1990) 239 [Sov. J. Nucl. Phys. **52** (1990) 126];
P. Ball, JHEP **9901** (1999) 010 [hep-ph/9812375].
- [11] P. Ball *et al.*, Nucl. Phys. B **529** (1998) 323 [arXiv:hep-ph/9802299];
P. Ball and V. M. Braun, Nucl. Phys. B **543** (1999) 201 [arXiv:hep-ph/9810475].
- [12] J. Gronberg *et al.* [CLEO Collaboration], Phys. Rev. D **57** (1998) 33 [arXiv:hep-ex/9707031].
- [13] G. Martinelli and C. T. Sachrajda, Phys. Lett. B **190** (1987) 151;
T. A. DeGrand and R. D. Loft, Phys. Rev. D **38** (1988) 954;
D. Daniel, R. Gupta and D. G. Richards, Phys. Rev. D **43** (1991) 3715.
- [14] L. Del Debbio *et al.* [UKQCD Collaboration], Nucl. Phys. Proc. Suppl. **83** (2000) 235
[arXiv:hep-lat/9909147];
L. Del Debbio, M. Di Pierro and A. Dougall, Nucl. Phys. Proc. Suppl. **119** (2003) 416
[arXiv:hep-lat/0211037].
- [15] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989) [Sov. J. Nucl. Phys. **50**, 511 (1989)].
- [16] P. Ball and M. Boglione, Phys. Rev. D **68** (2003) 094006 [arXiv:hep-ph/0307337].
- [17] P. Ball and V. M. Braun, Phys. Rev. D **54** (1996) 2182 [arXiv:hep-ph/9602323].
- [18] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45** (1992) 1754.
- [19] A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, Phys. Lett. B **508** (2001) 279
[Erratum-ibid. B **590** (2004) 309] [arXiv:hep-ph/0103119].
- [20] I. I. Bigi, arXiv:hep-ph/0501084.
- [21] A. Schmedding and O. I. Yakovlev, Phys. Rev. D **62** (2000) 116002 [arXiv:hep-ph/9905392].
- [22] A. P. Bakulev *et al.*, Phys. Rev. D **70** (2004) 033014 [Erratum-ibid. D **70** (2004) 079906]
[arXiv:hep-ph/0405062].
- [23] D. J. Gross and F. Wilczek, Phys. Rev. D **9** (1974) 980;
M. A. Shifman and M. I. Vysotsky, Nucl. Phys. B **186**, 475 (1981).
- [24] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B **345** (1990) 137.
- [25] For instance:
V. Barger *et al.*, Phys. Lett. B **598** (2004) 218 [arXiv:hep-ph/0406126];
S. Khalil and E. Kou, arXiv:hep-ph/0407284;
S. Mishima and T. Yoshikawa, Phys. Rev. D **70** (2004) 094024 [arXiv:hep-ph/0408090];
A. J. Buras *et al.*, arXiv:hep-ph/0410407;
S. Baek *et al.*, arXiv:hep-ph/0412086.
- [26] T. Feldmann and T. Hurth, JHEP **0411** (2004) 037 [arXiv:hep-ph/0408188].
- [27] M. Bona *et al.* [UTfit Collaboration], arXiv:hep-ph/0501199.

- [28] B. Aubert *et al.* [BABAR Collaboration], [arXiv:hep-ex/0408089];
K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **93** (2004) 021601
[arXiv:hep-ex/0401029].
- [29] H. F. A. Group, arXiv:hep-ex/0412073.
- [30] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **93** (2004) 131801
[arXiv:hep-ex/0407057];
Y. Chao *et al.* [Belle Collaboration], Phys. Rev. Lett. **93** (2004) 191802
[arXiv:hep-ex/0408100].
- [31] V. M. Belyaev *et al.*, Phys. Rev. D **51** (1995) 6177 [arXiv:hep-ph/9410280];
A. Khodjamirian *et al.*, Phys. Lett. B **457** (1999) 245 [arXiv:hep-ph/9903421].
- [32] Z. H. Li *et al.*, Phys. Rev. D **65** (2002) 076005 [arXiv:hep-ph/0208168].
- [33] A. J. Buras *et al.*, Nucl. Phys. B **697** (2004) 133 [arXiv:hep-ph/0402112].